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# INFLUENCE OF CAPILLARY FORCES ON THE REGIME OF WATER EVAPORATION IN HIGH-TEMPERATURE ROCKS

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*A one-dimensional problem of the injection of water into a geothermal reservoir saturated with a superheated vapor in the presence of capillary forces has been investigated in an isothermal approximation. It is shown that in wettable rocks the capillary forces increase the velocity of motion of the front and decrease it in unwettable ones. If the capillary forces play a decisive role, then in wettable rocks an extended region of phase transitions is formed behind the evaporation front under the action of capillary forces.*

**Introduction.** It was found in [1] that injection of water into a geothermal reservoir is accompanied by the formation of a narrow evaporation front. In a mathematical description the narrow front is modeled by an unknown mobile boundary of phase transition, and this allowed us in what follows to apply the analytical approach developed in [2–8]. In the limiting case, where the regime of flow is close to a piston one, simple formulas were obtained for the description of the basic parameters of flow, whereas their comparison with the data of the numerical calculation carried out with the aid of the TOUGH2 package of programs has demonstrated good agreement of the results [8].

The influence of capillary pressure on heat and mass transfer in geothermal systems was studied in [9–13]. The stationary problem of water evaporation in a bed under the action of a temperature gradient was investigated in [9]. In [10, 11], the results of numerical simulation are presented, which make it possible to evaluate the role of the TOUGH2 package of programs. The mathematical model of extraction of vapor from a water-saturated bed, which takes into account the capillary forces at the evaporation front that separates single-phase regions, was formulated in [12, 13]. In a one-dimensional nonstationary case an asymptotic solution of the problem was obtained that admits a simple analysis of the influence of capillary forces on extraction of vapor from a bed.

In the present work, the mathematical model that describes the action of capillary forces in extraction of vapor from a high-temperature reservoir [12, 13] is extended to the problem of water injection into a geothermal reservoir saturated with a superheated vapor. The temperature drop at the evaporation front depends on the intensity of phase transition and, as was found, it is small in a wide range of parameters [8]. With insignificant changes in temperature the surface tension changes little, and the influence of temperature on the capillary pressure magnitude can be neglected. By virtue of the fact that in the front statement capillary forces act only on an interface, it is interesting to consider the case of isothermal injection. Such a situation is encountered, e.g., when water moves in a geothermal reservoir from the region of a high pressure saturated with water into the region of low pressure saturated with vapor or when water is pumped into a geothermal reservoir located at a considerable depth, when water is heated appreciably in the well that passes through high-temperature impermeable rocks. In this case, the temperature in the bed changes only as a result of evaporation, and this change is insignificant, which makes it possible to consider an isothermal approximation. This approximation was also used in analyzing the precipitation of salts on evaporation in a geothermal reservoir [14], when a transverse heat flux in a thin permeable bed suppresses a change in temperature.

In the present work, the problem of injection of water into a geothermal bed saturated with a superheated vapor is considered. In an isothermal approximation the influence of capillary forces on the motion of phases is investigated. The solution of the problem in a self-similar approximation was found. An approximate formula for the velocity of motion of the phase interface in wettable rocks was obtained, when the velocity is determined in the main by

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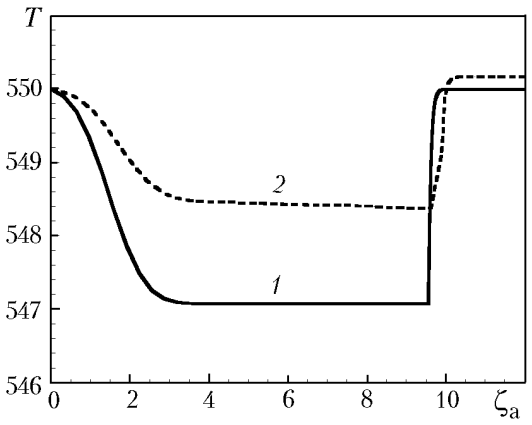



Fig. 1. Characteristic distribution of temperature as a function of the self-similar variable: 1) self-similar solution; 2) results of numerical simulation. $T_0 = T^0 = 550$ K, $P^0 = 8 \cdot 10^6$ Pa. $\zeta_a = \zeta\sqrt{\frac{\kappa_2}{a}}$, $a = 10^{-6}$ m/sec$^2$. $T$, K.

capillary forces. It is shown that in this case the presence of capillary forces leads to the formation of an extended evaporation region.

**Formulation of the Problem.** We will consider the motion of water in a geothermal reservoir from a region of high pressure into a region of low pressure saturated with a superheated vapor provided that the temperatures of the regions coincide. During motion into the region of vapor with a low pressure $P_0$ an evaporation surface is formed that separates the regions of water and vapor. During evaporation of the liquid phase absorption of heat occurs, which causes cooling of rocks. Figure 1 presents the results of analytical and numerical calculations of the injection of water; they were obtained within the framework of the mathematical model considered in [8], when the temperature of the pumped water coincides with the temperature of the geothermal reservoir. The temperature at the evaporation front has a minimum, which is due to the absorption of heat in phase transition. Behind the evaporation front, just as in the case of injection of a cold water into high-temperature rocks [1], an extended region of constant temperature is formed. Near the boundary $x = 0$, a boundary layer is formed due to conductive heat transfer, since the temperature is taken constant on this boundary. When the boundary pressure $P^0$ increases, which corresponds to the increase in the rate of liquid pumping, the boiling pressure $P_f$ also increases and the intensity of vapor generation decreases. As a result, the evaporation temperature $T_f$ tends to the initial value $T_0$. For the characteristic values of the parameters, the drop in temperature because of the absorption of heat on the evaporation front is small; therefore in a first approximation the process of pumping can be considered isothermal. The difference in the distributions of temperature for the considered regime of pumping is explained by the substantial difference between the initial pressure and vapor pressure at the evaporation front, so that the linear solution of the problem of vapor motion yields an excess flux of vapor mass at the front, which leads to a larger deviation of temperature from the initial value. When the evaporation pressure is close to the initial one, the results of analytical and numerical calculations coincide [8].

The above-given arguments point to the fact that for certain cases of flows in geothermal systems an isothermal approximation can be used which also describes well the process at small values of porosity and a small thickness of the bed, when a drop in temperature is compensated by an influx of heat from the surrounding rocks. Making use of this fact, we investigate the influence of capillary forces on the injection of water into a geothermal bed saturated with vapor, assuming the temperature constant, $T = T_0 = \text{const}$.

The isothermal motion of vapor in permeable rocks is described by the Darcy law:

$$v_v = -\frac{k}{\mu_v} \operatorname{grad} P, \tag{1}$$

by the mass conservation law:

$$\phi \frac{\partial}{\partial t} \rho_v + \operatorname{div} \rho_v v_v = 0 , \tag{2}$$

and by the equation of state for vapor:

$$P = \rho_v R T . \tag{3}$$

For a water-saturated region the Darcy law, the law of mass conservation, and the equation of state for water are also applicable. In [8] it was shown that the motion of water behind the evaporation front occurs in a quasi-stationary regime, and the water can be considered as an incompressible liquid, $\rho_w$ = const. Then the system of basic equations has the form

$$v_w = -\frac{k}{\mu_w} \operatorname{grad} P , \tag{4}$$

$$\operatorname{div} v_w = 0 . \tag{5}$$

During injection of water into a geothermal reservoir, a phase transition front is formed, which moves from the pumping well and separates the permeable bed into two regions: one saturated with water and the other saturated with vapor. The boundary conditions on the interface follow from the mass conservation law provided there is local thermodynamic equilibrium:

$$\phi \left( 1 - \frac{\rho_v}{\rho_w} \right) V_n = \frac{k \rho_v}{\mu_v \rho_w} (\operatorname{grad} P)_{n+} - \frac{k}{\mu_w} (\operatorname{grad} P)_{n-} . \tag{6}$$

The conservation law is complemented by the Clausius–Clapeyron equation, which determines the pressure of phase transition as a temperature function. The saturation pressure over the plane surface can be found from the relation

$$\log \frac{P_f}{P_a} = A + \frac{B}{T_f} , \tag{7}$$

where $A$ = 5.44; $B$ = −2005.1 K; $P_a$ = $10^5$ Pa.

In considering capillary forces in a porous medium, the relationship between the pressures of vapor and water is valid:

$$P_{w-} = P_{v+} + P_c , \tag{8}$$

where the capillary pressure $P_c$ is determined by the Laplace relation:

$$P_c = -\frac{2\sigma \cos\theta}{r} .$$

The characteristic dimension of a pore is assumed to be equal to $r = \sqrt{k/\varphi}$. Then the capillary force acting on the vapor–water interface is determined in terms of the porous medium parameters as

$$P_c = -4\sigma \sqrt{\frac{\phi}{k}} \cos\theta . \tag{9}$$

We note that the formula (9) determines a pressure jump across the vapor–water interface. If there is an extended region of vapor–water mixture, the capillary pressure equal to the difference between the pressures of water and vapor depends on the water saturation and is determined by the Laverette function.

If $P_*$ is the saturation pressure over a plane surface, then the saturation pressure over the meniscus can be found from the Kelvin formula:

$$P_{v+} = P_* \exp\left(-\frac{2\sigma V_w}{rR_0T}\right). \tag{10}$$

Using relations (7) and (10), we obtain a generalized Clausius–Clapeyron equation at the phase transition front; this equation couples the pressure and temperature of vapor generation, with the action of capillary forces taken into account:

$$P_{v+} = P_a \exp\left(2.3A + \frac{2.3B}{T_f} - \frac{2\sigma V_w}{rR_0T_f}\right)$$

or

$$P_{v+} = P_a \exp\left(2.3A + \frac{2.3B}{T_f} - \frac{4\sigma V_w}{R_0T_f}\sqrt{\frac{\phi}{k}}\cos\theta\right). \tag{11}$$

In the isothermal approximation we assume the phase change temperature to be equal to the initial one, $T_f = T_0$. Then the full system of conditions on the mobile evaporation boundary consists of relations (6), (8), and (11).

**Self-Similar Solution.** We will consider the solution of the problem in a one-dimensional nonstationary case. We will prescribe the boundary and initial conditions in the following way: $t = 0$: $X(0) = 0$, $P = P_0$; $X = 0$: $P = P^0$. As is shown in [8], in the region of water, $0 < x < X(t)$, a stationary equation for pressure is valid which follows from Eqs. (4) and (5):

$$\frac{d^2P}{dx^2} = 0\,. \tag{12}$$

In the region saturated with vapor, we assume that the saturation pressure at the boiling front differs little from the initial pressure of the bed. Applying the procedure of linearization to Eqs. (1)–(3), we obtain a linear equation in the region of vapor, $X(t) < x < \infty$:

$$\frac{\partial P}{\partial t} = \kappa_1 \frac{\partial^2 P}{\partial x^2}, \quad \kappa_1 = \frac{kP_0}{\phi\mu_v}. \tag{13}$$

The one-dimensional problem with constant initial and boundary conditions admits self-similar solutions of the form

$$T = T(\xi), \quad P = P(\xi), \quad \xi = x/2\sqrt{t}, \quad X(t) = \beta\sqrt{t}\,.$$

The characteristic scale of length of the formulated problem is determined by the quantity $\kappa_2 = kP^0/\varphi\mu_w$. We will introduce the dimensionless coordinate $\zeta = \xi/2\sqrt{\kappa_2}$ and the dimensionless self-similar velocity of the front $\gamma = \beta/2\sqrt{\kappa_2}$. In the region of vapor, $\gamma < \zeta < \infty$, the pressure distribution can be found from Eq. (13) and has the form

$$P(\zeta) = (P_{v+} - P_0)\frac{\operatorname{erfc}(\zeta\sqrt{\kappa_2/\kappa_1})}{\operatorname{erfc}(\gamma\sqrt{\kappa_2/\kappa_1})} + P_0\,. \tag{14}$$

In the region of water, $0 < \zeta < \gamma$, Eq. (12) yields the linear distribution of pressure:

$$P(\zeta) = P^0 + \left(P_{w-} - P^0\right)\frac{\zeta}{\gamma}. \tag{15}$$

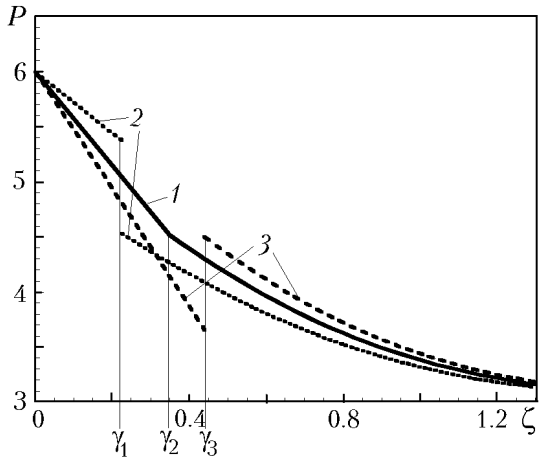



Fig. 2. Characteristic distribution of pressure as a function of the self-similar variable: 1) neutral rocks ($\theta = 90^o$); 2) wettable rocks ($\theta = 30^o$); 3) unwettable rocks ($\theta = 150^o$). $T$ = 530 K, $P^0 = 7\cdot10^6$ Pa. $P$, MPa.

Substituting solutions (14) and (15) into the system of conditions on the mobile evaporation boundary, we obtain a system of transcendental equations for the unknown quantities $P_{v+}$, $P_{w-}$, and $\gamma$:

$$\sqrt{\pi}\left(1-\frac{\rho_{v+}}{\rho_w}\right)\gamma-\frac{\sqrt{\pi}}{2\gamma}\left(\frac{P_{w-}}{P_0}-\frac{P^0}{P_0}\right)-\sqrt{\frac{\kappa_1}{\kappa_2}}\left(\frac{P_{v+}}{P_0}-1\right)\frac{\exp\left(-\gamma^2\sqrt{\kappa_2/\kappa_1}\right)}{\operatorname{erfc}\left(\gamma\sqrt{\kappa_2/\kappa_1}\right)}=0\,, \tag{16}$$

$$P_{v+}=P_a\exp\left(2.3A+\frac{2.3B}{T_0}-\frac{4\sigma V_w}{R_0T_0}\sqrt{\frac{\phi}{k}}\cos\theta\right),$$

$$P_{w-}=P_{v+}+P_c\,.$$

Here $\rho_{v+}=\dfrac{P_{v+}}{RT_0}$. The latter term in (16) describes the mass flux in the region before the front and depends mainly on the difference between the initial pressure and evaporation pressure. For fairly large injection rates, or when the initial state of vapor approaches the saturation state, a pumping regime close to a piston one is realized. In this case, the last term in the equation can be neglected, and for the dimensional self-similar velocity of the front motion we have

$$\beta=\sqrt{2\frac{k\,(P^0-P_{w*})}{\phi\mu_w}}=\sqrt{2\frac{k\,(P^0-P_{v+}-P_c)}{\phi\mu_w}}\,. \tag{17}$$

In the general case, the system of transcendental equations was solved numerically for the characteristic values of the thermodynamic parameters of water, vapor, and rocks [8]. In all of the cases, it was assumed that $k = 10^{-15}$ m$^2$, $\phi = 0.1$, and $P_0 = 3\cdot10^6$ Pa.

**Analysis of Results.** Figure 2 shows the characteristic form of pressure distribution in pumping of water into wettable, unwettable, and neutral rocks. In transition through the interface the pressure undergoes a jump equal to the magnitude of capillary pressure. For an unwettable medium the capillary pressure calculated from Eq. (9) is positive, and the pressure in the liquid phase is higher than the vapor pressure. It is seen that the positive capillary pressure at the front for the unwettable medium decreases the pressure gradient behind the front, and this leads to a decrease in the rate of pumping. In this case, even at a sufficiently high injection pressure $P^0$ the velocity of the front $\gamma$ can ap-

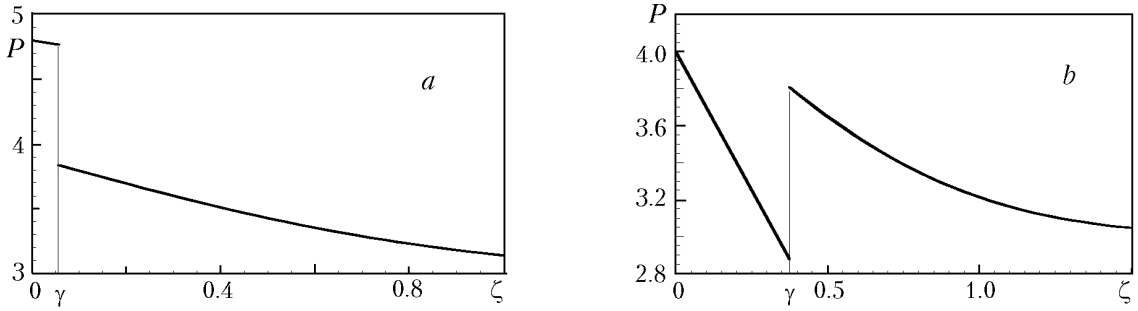



Fig. 3. Distribution of pressure for the unwettable medium (a) and wettable rocks (b): a) $\theta = 150^{o}$, $T = 530$ K, $P^0 = 4.8 \cdot 10^6$ Pa; b) 30, 520, and $4 \cdot 10^6$. $P$, MPa.

proach zero if $P^0 \to P_{v+} + P_c$ (Fig. 3a). If $P_{v+} + P_c > P^0$, there is no solution of the problem, which corresponds to the impossibility of pumping water into the bed.

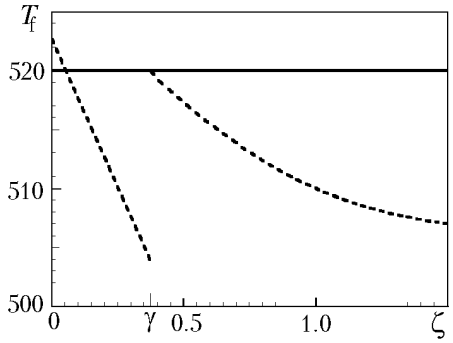
However, in the case of a wettable porous medium the velocity of the front can be considerable at small injection pressures (Fig. 3b). Here, the capillary forces increase the pressure gradient in the region of water and intensify the process of pumping. At considerable capillary forces and small initial and boundary pressures a regime of flow, which is usually called the suction regime, is realized. If the initial pressures of vapor and of water in a well are the values of one order of magnitude, $P_0 \sim P_{v*} \sim P^0$, then the suction regime is realized near the equilibrium position (when the vapor pressure is lower than the pressure of phase transition and the pressure of the liquid exceeds it slightly) in such a way as to satisfy the thermodynamic conditions of the existence of vapor and water. Using formula (17), for the indicated case we obtain

$$\beta = \sqrt{2\frac{k\,(P^0 - P_{w*})}{\phi\mu_w}} \approx \sqrt{2\frac{k\,(-P_c)}{\phi\mu_w}} = \left(8\frac{\sigma\cos\theta}{\mu_w}\right)^{\frac{1}{2}}\left(\frac{k}{\phi}\right)^{\frac{1}{4}},$$

i.e., the rate of water suction due to the capillary pressure is proportional to the fourth-degree root of the permeability.

As shown in Fig. 3b, in the case of wettable rocks the capillary forces lead to a reduction of pressure in the liquid behind the front. We will consider the distribution of the phase transition temperature $T_f$ in the regions saturated with water and vapor. The phase change temperature is calculated from the generalized Clausius–Clapeyron formula (11) from the distribution of water and vapor found, respectively (Fig. 4). It is seen that in the region of vapor the phase change temperature is lower than the local temperature, and the vapor is in a superheated state. Behind the phase change front, in the region adjacent to the evaporation surface, the temperature of water is higher than the phase change temperature, $T_w > T_f$, i.e., the water is in a superheated state, which contradicts the assumption on the existence of a liquid phase. This means that instead of the evaporation front, where the entire liquid phase is converted into a gaseous state, a more complex configuration is formed that consists of the partial phase transition surface and of the extended evaporation region in which vapor and water are in a state of thermodynamic equilibrium. In the narrow region modeled by the front of partial phase transition only a certain portion of the liquid phase is converted into vapor, whereas the remaining part is evaporated in the extended region. In this case, there exists an analogy with the appearance of a two-phase zone, when vapor is extracted from a highly permeable water-saturated bed [15]. The intensity of phase transitions in the extended region is characterized by the extent of water overheating behind the front. If the temperature of water calculated from the front model is substantially higher than the phase change temperature, then in the two-phase zone the volumetric fraction of vapor is commensurable with the water saturation $S$. If the deviation from the equilibrium state is inconsiderable, then in the extended evaporation zone being formed the value of the water saturation $S$ is close to unity. For the case presented in Fig. 4, the maximum deviation amounts only to 15 K. On deviation from the equilibrium state by several degrees the formation of a small amount of vapor compensates for the superheating of water.

Figure 5 demonstrates the distribution of the function of water saturation obtained in numerical simulation with the use of the TOUGH2 package at the same values of parameters. It is seen that a sharp front of a partial phase



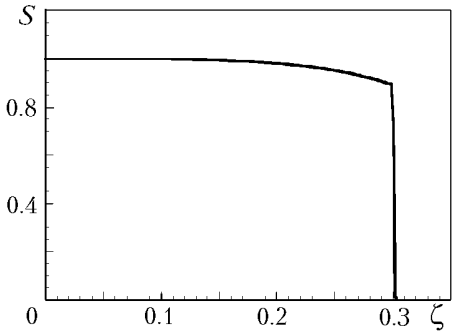




Fig. 4. Distribution of the phase change temperature (dashed lines) as a function of local pressure; the solid line corresponds to the constant temperature of the bed. The parameters are same as in Fig. 3b. $T_f$, K.

Fig. 5. Distribution of the water saturation function obtained by numerical simulation at $\theta = 30^o$, $T = 520$ K, and $P^0 = 4 \cdot 10^6$ Pa.

transition is formed, where the water saturation is changed by a jump from zero to a certain value close to unity. Behind the front the water saturation changes little and differs from unity most noticeably only near the front, where, according to the front model, the highest overheating of water is observed.

In comparison with the analytical solution obtained for the front model, a numerical solution yields a smaller value for the velocity of the phase transition front and the difference amounts to about 20%. This is explained by the appearance of the extended region of phase transition containing vapor. The compressibility of the vapor–water mixture is determined by the compressibility of vapor, and the vapor–water mixture exerts a higher resistance in pumping into the bed, which precisely explains the delay of the front in the numerical solution. If the injection pressure appreciably exceeds the capillary pressure, the overeating is observed only in the narrow region behind the front. In this case, the dimensions of the zone of the mixture being formed are small, its presence does not exert a noticeable effect on the model of flow, and the front solution is a rather good approximation. For an unwettable or neutral medium the region of a two-phase state is not formed at all, and the velocity of the front can be found with a high degree of accuracy from the front solution (see Fig. 1).

**Conclusions.** The mathematical model presented in this work for injection of cold water into a geothermal reservoir in an isothermal approximation allows one to obtain an analytical solution which yields the distribution of pressure and the phase change front velocity. The accuracy of the obtained simple estimates of the front motion velocity is confirmed by the results of numerical simulation with the use of the TOUGH2 package. The data of calculations of the injection of a cold water into wettable rocks point to the overheating of water behind the evaporation front and correspondingly to the disturbance of the condition of thermodynamic equilibrium in the front model. We can draw an analogy with the classical model of crystallization of a binary melt, where overcooling of the melt before the crystallization front appears because of the presence of an impurity. Here, the disturbance of the condition of thermodynamic equilibrium appears behind the front, which is expressed in the overheating of water due to the action of capillary forces. Similarly, just as in the problem of crystallization of a melt, the overheating of water initiates the formation of an extended region of phase changes. This fact is confirmed by a direct numerical experiment with the aid of the TOUGH2 package of programs. The formation of the mixture zone characterized by a higher compressibility leads to a decrease in the rate of water pumping into the geothermal reservoir.

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## NOTATION

$k$, permeability, $m^2$; $P$, pressure, Pa; $P_a$, atmospheric pressure, Pa; $r$, characteristic dimension of a pore, m; $R$, gas constant for vapor, J/(kg·K); $R_0$, universal gas constant, J/(mole·K); $S$, water saturation; $t$, time, sec; $T$, tempera-

ture, K; $v$, filtration rate, m/sec; $V$, velocity of the phase change front, m/sec; $V_w$, molar volume of water, $m^3$/mole; $x$, coordinate, m; $X(t)$, law of evaporation front motion, m; $\phi$, porosity; $\mu$, viscosity, Pa·sec; $\theta$, angle of wetting, deg; $\rho$, density, $kg/m^3$; $\sigma$, coefficient of surface tension, $J/m^2$. Subscripts: a, atmospheric; f, phase transition; v, vapor; c, capillary; w, water; n, normal to the front; 0, initial value. Superscript: 0, boundary value. The signs "–" and "+" denote the values behind and before the front.

## REFERENCES


1. C. Calore, K. Pruess, and R. Celati, Modeling studies of cold water injection into fluid-depleted, vapor-dominated geothermal reservoirs, in: *Proc. 11th Workshop on Geothermal Reservoirs Engineering*, Stanford, CA, USA (1986), pp. 161–168.
2. G. Bodvarsson, Thermal problems in the siting of reinjection wells, *Geothermics*, **1**, No. 2, 63–66 (1972).
3. M. J. O'Sullivan and K. Pruess, Analysis of injection testing of geothermal reservoirs, *Trans. Geoth. Resour. Council*, **4**, 401–405 (1980).
4. K. Pruess, C. Calore, R. Celati, and Y. S. Wu, An analytical solution for heat transfer at a boiling front moving through a porous medium, *Int. J. Heat Mass Transfer*, **30**, No. 12, 2595–2602 (1987).
5. A. A. Barmin and G. G. Tsypkin, On motion of the phase change front during injection of water into a hydrothermal vapor-saturated bed, *Dokl. Ross. Akad. Nauk*, **350**, No. 2, 195–197 (1996).
6. A. W. Woods and S. D. Fitzgerald, The vaporization of a liquid front moving through a hot porous rock. Pt. II. Slow injection, *J. Fluid Mech.*, **343**, 303–316 (1997).
7. A. W. Woods, Liquid and vapour flow in superheated rock, *Ann. Rev. Fluid Mech.*, **31**, 171–199 (1999).
8. G. G. Tsypkin, C. Calore, and M. Markolini, Mathematical simulation of the injection of cold water into a depleted high-temperature geothermal reservoir, *Teplofiz. Vys. Temp.*, **44**, No. 3, 453–459 (2006).
9. K. S. Udell, Heat transfer in porous media: considering phase change and capillarity — the heat pipe effect, *Int. J. Heat Mass Transfer*, **28**, 485–496 (1985).
10. K. Pruess, Grid orientation and capillary pressure effects in the simulation of water injection onto depleted vapor zone, *Geothermics*, **20**, 257–277 (1991).
11. K. Pruess and M. J. O'Sullivan, Effects of capillarity and vapor adsorption in the depletion of vapor-dominated geothermal reservoirs, in: *Proc. 17th Workshop on Geothermal Reservoirs Engineering*, Stanford, CA, USA (1992), pp. 165–174.
12. G. G. Tsypkin and C. Calore, A mathematical model of water–vapor phase transitions in geothermal systems in the presence of capillary forces, *Dokl. Ross. Akad. Nauk*, **385**, No. 2, 177–180 (2002).
13. G. G. Tsypkin and C. Calore, Role of capillary forces in vapour extraction from low-permeability, water–saturated geothermal reservoir, *Geothermics*, **32**, 219–237 (2003).
14. G. G. Tsypkin and A. W. Woods, Precipitate formation in a porous rock through evaporation of saline water, *J. Fluid Mech.*, **537**, 35–53 (2005).
15. G. G. Tsypkin and A. W. Woods, Vapour extraction from a water saturated geothermal reservoir, *J. Fluid Mech.*, **506**, 315–330 (2004).